Cluster mean-field model of the spin glasses: static properties

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We propose and solve a phenomenological model of the spin-glass alloys in which dynamical clusters are the basic entity. Intracluster interactions are treated exactly while intercluster terms are handled using the Edwards-Anderson (EA) random-mean-field theory. Numerical computations for average cluster sizes containing $3 \leq N \leq 6$ atoms of the static susceptibility and specific heat C_m show that the former exhibits a sharp cusp at the freezing temperature T_c. For antiferromagnetic intracluster interactions the latter has a rounded maximum at a slightly higher temperature which reflects the intracluster exchange constant. These features, both of which could not be explained previously, are in semiquantitative accord with experiment. For ferromagnetic clusters, agreement between theory and experiment for C_m is also satisfactory, but for somewhat larger ($N \geq 12$) cluster sizes. The present model represents a first step in going beyond a simple mean field theoretic approach; it can, thus, be inferred that the most important fluctuation corrections derive from strong correlations between nearby spins (clusters) to which correlations the specific heat is more sensitive than is the susceptibility.

I. INTRODUCTION

Recently, we proposed[1] a simple cluster-mean-field theory (CMFT) of the spin glasses, in which correlated clusters rather than individual spins are the basic entity. The motivations for the model were (i) to incorporate the important effects[2] of strongly interacting spin clusters into a simple mean-field theory of the spin glasses, and (ii) to suggest a mechanism which would lead to a rounded maximum in the specific heat (at a temperature T_0 above the freezing temperature T_c) and which at the same time would not affect the cusp in the magnetic susceptibility. This paper is a detailed exposition of the CMFT as sketched in a previous letter.[1]

Spin glasses are random dilute magnetic alloys such as AuFe and CuMn with impurity concentrations which lie between the Kondo and long-range magnetically ordered regimes. Measurements[3] of the static-zero-field susceptibility[4] χ, Mössbauer splitting,[5] anomalous Hall effect,[6] and muon precession rate[7] suggest that these alloys exhibit a sharp phase transition at a freezing temperature T_c. On the other hand, thermoelectric-power,[8] resistivity,[9] and magnetic-specific-heat[10] C_m measurements do not reveal any anomalies at T_c. Moreover, C_m generally has a broad maximum at a temperature $T_0 \sim (1.2\text{–}1.4)T_c$.

Much of the current theoretical effort[11–18] devoted to the spin glasses has been based on the Edwards-Anderson (EA) model[12] in which the spin glass is represented by a Heisenberg Hamiltonian with random, Gaussian distributed exchange interactions. The free energy of this quenched random system is obtained using the replica method.[12–14] This approach formally defines an order parameter $q = \langle \vec{S}_i^{\alpha} \cdot \vec{S}_i^{\beta} \rangle$ where α and β denote different replicas of the system and S_i is the spin at the ith site. Mean-field theoretic approximations to the free energy lead to a self-consistent equation for q which suggests the natural interpretation of this quantity as a random anisotropy field. These approaches yield expressions for C_m and χ in which *both* exhibit sharp cusps at a transition temperature T_c, in disagreement with experiment.[4,10] Similar difficulties arise with the exactly soluble models[16,18] and the "mean-random-field approach"[19] and its extensions.[20] These inconsistencies with the data have been attributed to inadequacies of both the replica method[13] and of mean-field theory.[21] However, mean-field theoretic approaches to the problem which did not utilize the replica "trick" have led to results very similar to those in which it was applied.[22] Renormalization-group calculations[21] may explain the absence of any observed anomaly in C_m at T_c but they also incorrectly predict that χ will *not* exhibit a cusp at the freezing temperature.

The notion that clusters of spins may play an important role in the spin glasses was proposed by Beck[23] and by Tholence and Tournier.[24] Experimental evidence for the importance of spin clusters is summarized in detail in Ref. 2. The authors of Ref. 24 viewed the alloy as composed of rigid giant moments frozen in by local magnetic anisotropy fields, whose microscopic origin was unspecified. The qualitative picture that emerged was that χ would have a maximum rather than a sharp cusp at T_c. Smith[25] adopted a more quantitative percolation theoretic approach based on correlated clusters of spins, to suggest a mechanism for obtaining a sharp cusp in χ. However, the behavior of C_m near T_c in his theory is apparently indeterminate.

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The present viewpoint of the spin glass is basically a synthesis of the EA mean-field theoretic approach and of the cluster theories. Spin clusters are viewed as units of spins which are tightly correlated, as a consequence, presumably of their spatial proximity. These units are present whether or not there is actual chemical clustering. Because the spins within a cluster interact strongly, intracluster interactions are treated exactly, while the weaker intercluster interactions can be approximated using mean-field theory. This decomposition is clearly possible in any dilute *random* system (even in the absence of chemical clustering) and also serves as a useful starting point for treating alloys which undergo the more usual types of magnetic long-range order.[26] A somewhat different way of treating these clustering effects was introduced by Reiss[27] and also by Larsen.[27] These authors postulated that even above T_c there would be nonvanishing internal fields (deriving from fluctuation effects) in the spin-glass alloy. In order to study the behavior of χ and C_m over an extended range of temperatures, it is important to consider the internal degrees of freedom of the cluster. Therefore, in contrast to previous[2,24] rigid-cluster theories, here we treat the cluster dynamics fully. The present approach can be viewed as a first step towards including fluctuation effects into the random-mean-field theory of the spin glasses. It can alternatively be justified on purely phenomenological grounds. As discussed in Ref. 1 it was found that for a range of average cluster sizes $N(3 \leq N \leq 6)$ and for antiferromagnetic intracluster interactions, the CMFT yielded a sharp cusp in χ at T_c and a rounded maximum in C_m at a higher temperature T_0 (which was characteristic of the intracluster exchange energy J^{intra}). As noted above, both these features are consistent with experiment. A small feature in C_m at T_c was also observed which may possibly be washed out when fluctuation effects are more correctly included. This behavior of C_m, which could not previously be explained, was also found using Monte Carlo calculations.[2] The present approach offers a simple physical explanation of results obtained from these numerical simulations. For ferromagnetic intracluster interactions, we expect that agreement with experiment is also satisfactory for larger ($N \geq 12$) cluster sizes. This expectation is borne out in Ising-model calculations, which will be discussed in a future paper.

The CMFT free energy is evaluated using the replica method. In contrast to the usual mean-field[11] theory, we introduce two variational parameters, the (temperature dependent) cluster moment M as well as the anisotropy field q. While the shortcomings of the replica method at $T \sim 0$, are well known,[13,14] most of the interest here is at temperatures near and above T_c where it is expected to be a reasonable approach. As in the case of simple EA mean-field theory,[12] the $q=0$ extrema of the free energy do not (in the limit that the number of replicas n approaches zero) correspond to minima but are instead saddle points in (M,q) space.

The remainder of the paper is divided into two sections. In Sec. II we describe the model Hamiltonian; the free energy is obtained and the two coupled integral equations for the variational parameters derived. The specific heat and spin susceptibility are calculated in terms of q and M. In Sec. III numerical results for χ, C_m, q, and M for different average cluster sizes and for both antiferromagnetic and ferromagnetic intracluster interactions are presented.

II. GENERAL THEORETICAL RESULTS

The phenomenological model Hamiltonian which serves as the starting point for the cluster-mean-field theory is

$$\mathcal{H} = -\sum_{\nu<\lambda=1}^{N_{\text{cl}}} J_{\nu\lambda}\vec{S}_\nu\cdot\vec{S}_\lambda - \sum_{\nu=1}^{N_{\text{cl}}}\sum_{i<j=1}^{N} J_{ij}^0\vec{S}_{i\nu}\cdot\vec{S}_{j\nu}\,, \qquad (2.1)$$

where $S_{i\nu}$ denotes the spin at the site i in the νth cluster and $\vec{S}_\nu = \sum_{i=1}^{N}\vec{S}_{i\nu}$. In Eq. (2.1) and what follows Greek indices refer to a particular cluster and Roman indices to a given spin within that cluster. J_{ij}^0 is the intracluster exchange interaction between the ith and jth spins and N is the average size of a cluster; these two parameters are variable and viewed as phenomenologically determined. The number of clusters N^{cl} is then fixed in terms of N and the impurity concentration c. The intercluster interactions $J_{\nu\lambda}$ are distributed according to the Gaussian formula

$$P(J_{\nu\lambda}) = 1/[(2\pi)^{1/2}J]\exp(-J_{\nu\lambda}^2/2J^2) \qquad (2.2)$$

where $J_{\nu\lambda}$ is nonzero only if ν represents one of the z cluster near neighbors of the λth cluster. Because the clusters are assumed to be far apart, compared to their average size, $J_{\nu\lambda}$ can be taken to be independent of the location of the spins within a cluster. A rather important simplifying feature of mean-field theoretic solutions of the model Hamiltonian [Eq. (2.1)] is that each cluster may be viewed as having the same size and $\{J_{ij}^0\}$ corresponding to the average cluster in the alloy. This follows because in mean-field theory the clusters are effectively averaged over to obtain the mean (random anisotropy) field; the distribution in the

size of averaged-over spins is not of particular significance. This will be discussed in more detail below. In order for the cluster concept to make sense, it follows that the intracluster exchange interactions are necessarily stronger than the intercluster terms.

The free energy is evaluated using the replica method

$$F=-kT\lim_{n\to 0}\frac{1}{n}\left[\int\prod dJ_{\nu\lambda}P(J_{\nu\lambda})\,\mathrm{Tr}_n\exp\left(\frac{1}{kT}\sum_{\alpha=1}^{n}\sum_{\nu<\lambda}J_{\nu\lambda}\vec{S}_\nu^\alpha\cdot\vec{S}_\lambda^\alpha-\frac{1}{kT}\sum_{\alpha=1}^{n}\sum_{\nu}\mathcal{H}_{\mathrm{cl}}^{\alpha\nu}\right)-1\right], \tag{2.3}$$

where the intracluster Hamiltonian for the αth replica is

$$\mathcal{H}_{\mathrm{cl}}^{\alpha\nu}=-\sum_{i<j=1}^{N}J_{ij}^0\vec{S}_{i\nu}^\alpha\circ\vec{S}_{j\nu}^\alpha\,. \tag{2.4}$$

Here Tr_n is the trace evaluated for the n replicas of the system. Performing the integration over $J_{\nu\lambda}$ yields

$$F=-kT\lim_{n\to 0}\frac{1}{n}\left[\mathrm{Tr}_n\exp\left(\sum_{\nu<\lambda}\frac{J^2}{2(kT)^2}\sum_\alpha\vec{S}_\nu^\alpha\circ\vec{S}_\lambda^\alpha\sum_\beta\vec{S}_\nu^\beta\circ\vec{S}_\lambda^\beta-\frac{1}{kT}\sum_\alpha\sum_\nu\mathcal{H}_{\mathrm{cl}}^{\alpha\nu}\right)-1\right]. \tag{2.5}$$

In deriving Eq. (2.5) we have set commutators like $[S_\nu^\alpha\cdot S_\lambda^\alpha, S_{i\nu}^\alpha\cdot S_{j\mu}^\alpha]=0$. This is consistent with the mean-field decoupling we now apply to Eq. (2.5). We define the two variational parameters

$$q\equiv\langle\vec{S}_\nu^\alpha\cdot\vec{S}_\nu^\beta\rangle\,,\quad\alpha\neq\beta \tag{2.6a}$$

and

$$M=\langle\vec{S}_\nu^\alpha\cdot\vec{S}_\nu^\alpha\rangle \tag{2.6b}$$

The first is analogous to the usual EA spin-glass order parameter; it differs only in that S_ν denotes the cluster spin rather than the spin of one impurity atom only. Here M is the expectation value of the magnitude of the cluster spin. Using

$$\langle\vec{S}_\nu^\alpha\circ\vec{S}_\lambda^\beta\rangle=\delta_{\nu\lambda}[\delta_{\alpha\beta}M+(1-\delta_{\alpha\beta})q]\,, \tag{2.7}$$

we find

$$F=-kT\lim_{n\to 0}\frac{1}{n}\left\{\mathrm{Tr}_n\exp\left[\frac{\bar{J}^2}{2(kT)^2}\sum_\nu\left(\tfrac{1}{3}M\sum_\alpha\vec{S}_\nu^\alpha\cdot\vec{S}_\nu^\alpha+\tfrac{1}{3}q\sum_{\alpha\neq\beta}\vec{S}_\nu^\alpha\cdot\vec{S}_\nu^\beta-\tfrac{1}{6}n[M^2+(n-1)q^2]\right)-\frac{1}{kT}\sum_\alpha\sum_\nu\mathcal{H}_{\mathrm{cl}}^{\alpha\nu}\right]-1\right\}, \tag{2.8}$$

where $\bar{J}\equiv\sqrt{z}J$. Taking the $n\to 0$ limit and introducing an integration variable r as in Ref. 12 we have

$$F(q,M)=-N^{\mathrm{cl}}kT\left(\frac{\bar{J}^2}{12(kT)^2}(q^2-M^2)+(2\pi)^{-3/2}\int d^3r e^{-r^2/2}\times\ln\mathrm{Tr}\exp(-\beta H^{\mathrm{eff}})\right) \tag{2.9}$$

and

$$-\mathcal{H}^{\mathrm{eff}}=\sum_{i<j}J_{ij}^0\vec{S}_{i\nu}\cdot\vec{S}_{j\nu}+\bar{J}(\tfrac{1}{3}q)^{1/2}\vec{r}\cdot\vec{S}_\nu+\frac{\bar{J}^2}{6kT}(M-q)\vec{S}_\nu\cdot\vec{S}_\nu \tag{2.10}$$

is the effective cluster Hamiltonian and we now define $\beta\equiv 1/kT$.

In deriving Eq. (2.10), we have used the fact that all three terms in Eq. (2.10) commute with one another. The physical interpretations of the various contributions to $\mathcal{H}^{\mathrm{eff}}$ are as follows. The first term is the intracluster interaction; the second represents the interactions of the random anisotropy field with the spins in the cluster. This field arises because of intercluster interactions and its strength is given in terms of the order parameter q. The third term is an effective ferromagnetic intracluster interaction. In the usual EA mean-field theory this is a c number and is combined with the first term in Eq. (2.9). This term, which cannot be as easily motivated on physical grounds, will be discussed in more detail in Sec. III.

The parameters M and q are derived from the variational conditions $[\partial F(q,M)/\partial q]_M=0$ and $[\partial F(q,M)/\partial M]_q=0$. This yields

$$M=(2\pi)^{-3/2}\int d^3r\,e^{-r^2/2}\,\mathrm{Tr}(\vec{S}_\nu\cdot\vec{S}_\nu e^{-\beta H^{\mathrm{eff}}})/Z \tag{2.11}$$

and

$$M-q=(2\pi)^{-3/2}\int d^3r\, e^{-r^2/2}(3/q)^{1/2}(\beta\bar{J})^{-1} \times \mathrm{Tr}(\vec{S}_\nu\cdot\vec{r}e^{-\beta H^{\mathrm{eff}}})/Z \qquad (2.12)$$

where

$$Z=\mathrm{Tr}e^{-\beta H^{\mathrm{eff}}}. \qquad (2.13)$$

When a distribution of cluster sizes is included, we find that the right-hand sides of the self-consistency conditions [Eqs. (2.11) and (2.12)] are modified to include an overall factor $\sum_{N'}P(N')$ where $P(N')$ is the probability that the cluster will contain N' atoms. The precise solution of these equations for arbitrary $P(N')$ is difficult to obtain. However, we have compared our approximation $P(N')\approx\delta(N'-N)$, where N is the average number of spins per cluster, with more exact numerical solutions for several different $P(N')$ in the case of a random *ferromagnet*. In this simpler case we find that a distribution in sizes of the cluster has no qualitative significance and, therefore, anticipate that the same result will be valid for the spin glass system.

It should be noted that Eqs. (2.11) and (2.12) are physically very plausible and could essentially have been predicted (aside from numerical prefactors) on purely phenomenological grounds. The first equation represents the expectation value of the cluster moment when the cluster is subjected to a random magnetic anisotropy field. The second yields the mean value of the random field and is closely analogous to the usual Weiss mean-field self-consistency equation. In view of these remarks it is reasonable to conclude that despite difficulties associated with the replica method[13] and the identification of the correct order parameter[2] much of the essential physics of a spin-glass alloy must be contained in Eqs. (2.11) and (2.12).

Note that it follows from Eq. (2.11) that the cluster moment is temperature dependent; at temperatures high compared to the intracluster exchange interactions $M\to NS(S+1)$ where $S=\frac{1}{2}$ is the spin of a single impurity atom. As $T\to 0$ the cluster moment has its ground-state value [$\frac{3}{4}$ or 0 for perfectly compensated antiferromagnets and $NS(NS+1)$ for ferromagnets]. A commonly used alternative physical picture[24,25] of the clusters in a spin glass, is to view them as having a temperature-dependent size. The present description of the size of the correlated clusters, as derived from $M(T)$ is equivalent to this.

The specific heat C_m and static susceptibility χ per cluster can be computed in terms of M and q. We find

$$C_m=C_m^{\mathrm{inter}}+C_m^{\mathrm{intra}}, \qquad (2.14)$$

where the intercluster contribution to the specific heat is

$$C_m^{\mathrm{inter}}\equiv\frac{d}{dT}\left(\frac{\bar{J}^2}{6kT}(q^2-M^2)\right), \qquad (2.15)$$

and the intracluster contribution is

$$C_m^{\mathrm{intra}}\equiv\frac{d}{dT}\int d^3r\, e^{-r^2/2} \times\frac{\mathrm{Tr}(\mathcal{H}_{\mathrm{cl}}e^{-\beta H^{\mathrm{eff}}})}{Z(2\pi)^{3/2}}, \qquad (2.16)$$

where $\mathcal{H}_{\mathrm{cl}}$ is given by Eq. (2.4) without the α, ν superscripts. The intercluster term is very similar to that found in simple EA mean-field theory. This term has a sharp cusp at T_c. On the other hand, the intracluster contribution has virtually no structure at T_c but instead contains a rounded maximum at a slightly higher temperature T_0 which is characteristic of the size of the exchange interactions within a cluster. This first EA-like contribution is relatively insensitive to N for large N: following Ref. 13 at $T_c\sim\bar{J}S^2$, one finds

$$C_m^{\mathrm{inter}}\sim\bar{J}^2[S(S+1)]^2/T_c^2,$$

where S is the spin of the cluster. Thus for large S, C_m^{inter} at T_c is independent of S. On the other hand, the second term C_m^{intra} increases[28] with N. For sufficiently large N this rounded term will dominate the first. The value of N at which the two contributions are comparable is around 3 for an antiferromagnetic and 12 for a ferromagnetic cluster. At low T the intracluster contribution varies as $e^{-1/T}$ whereas C_m^{inter} is linear in T as is found experimentally.[10]

The static spin susceptibility χ per cluster can be derived from F [Eq. (2.9)] generalized to include a magnetic field

$$\chi=g^2\mu_B^2(M-q)/3kT. \qquad (2.17)$$

Because of the temperature dependence of M, χ will differ slightly from its value as obtained in EA mean-field theory. These effects are qualitatively greatest above T_c for the antiferromagnetic cluster and below T_c for the ferromagnetic case. However, in all the examples which we have studied numerically, the susceptibility maintains a sharp cusp at T_c, similar to that found in conventional mean-field theory. Thus, in contrast to C_m, there appear to be no qualitative features in χ associated with the intracluster interactions.

III. NUMERICAL RESULTS

In this section we present results for M, q, χ, and C_m as a function of temperature T for various N and both ferro- and antiferromagnetic intracluster interactions.

Equations (2.11) and (2.12) were solved numerically using an iteration scheme based on the $T=0$ solutions. To evaluate the traces, the Hamiltonian $\mathcal{H}^{\text{eff}}$ [Eq. (2.10)] was diagonalized. As basis states we used the direct product of the N single spin (S_z) states. The resulting matrix (of dimension $2^N \times 2^N$) can be decomposed into blocks which are separately diagonalized. This makes it possible, in principle, to consider up to $N=11$ spins in a cluster. The necessary integrals were carried out using standard machine subroutines.

For simplicity we considered only near-neighbor intracluster interactions with a characteristic exchange constant J^0. Our conclusions for given N were not particularly sensitive to the configuration of atoms in the cluster. We present here fairly typical results in terms of the ratio of the two exchange constants $R \equiv |\bar{J}/J^0|$. The magnitude of R was chosen to reproduce approximately the experimentally observed ratio T_c/T_0. It should be noted that for $J^0<0$, Eqs. (2.11) and (2.12) have no antiferromagnetic ground-state solutions when R is sufficiently large ($\geq R_c$). This results from the third term in Eq. (2.10) which, for large enough $\bar{J}$, forces the cluster into a ferromagnetic ground state. In the present model, it is assumed that the cluster energy levels are primarily determined by the presumably stronger intracluster interactions (J^0) rather than the intercluster terms ($\bar{J}$) so that we regard the case $R>R_c$ as unphysical.

In the inset of Fig. 1(a) are plotted the normalized temperature dependences of M and q for $N=3$ and $R=0.55$. This can be compared with the inset of Fig. 1(b) in which these variational parameters are plotted for the ferromagnetic case for the same parameters. The order parameter q decreases nearly linearly as a function of increasing temperature as in the EA model. While q is qualitatively the same for both signs of J^0, the cluster moment M increases monotonically in the antiferromagnetic and decreases monotonically in the ferromagnetic cluster. The static susceptibility divided by $\chi(T_c)$ and specific heat per cluster are plotted versus temperature for these same cluster parameters in Figs. 1(a) (antiferromagnet) and 1(b) (ferromagnet). Note that in the first case, while there is a small feature at T_c which may be washed out by fluctuation effects, the specific heat has a rounded maximum at a temperature $T_0 \sim 1.7T_c$; the susceptibility has a sharp cusp at T_c similar to that found in EA mean-field theory. The behavior of both χ and C_m is qualitatively consistent with experimental[4,10] measurements in the spin-glass alloys for this case. On the other hand, as shown in Fig. 1(b), for ferromagnetic clusters both χ and C_m exhibit a sharp cusp at T_c. The intracluster contribution to C_m is relatively small for this value of $N=3$,[28] so that the intercluster term dominates and the behavior of the specific heat and susceptibility is similar to what is found in EA mean-field theory. It should be noted that Reiss[27] has found results similar to those plotted in Fig. 1(a) using a Green's function decoupling technique which includes fluctuation effects. These fluctuations play a role similar to that of the intracluster interactions.

In Fig. 2 are plotted $\chi(T)/\chi(T_c)$ and C_m vs T/T_c for $N=5$ with $R=0.3$ for both antiferromagnetic (a) and ferromagnetic (b) intracluster interactions. The insets show the temperature dependence of M and q. Four of the spins in the cluster were placed in a straight line, the fifth was attached to the second spin in the line. Note that for both the ferromagnetic and antiferromagnetic cases, the results are essentially the same as for $N=3$ (Fig. 1). Note also from Fig. 2(b) that because M increases more rapidly with temperature in the interval $0 \leq T \leq T_c$ than for $N=3$, the susceptibility cusp is slightly

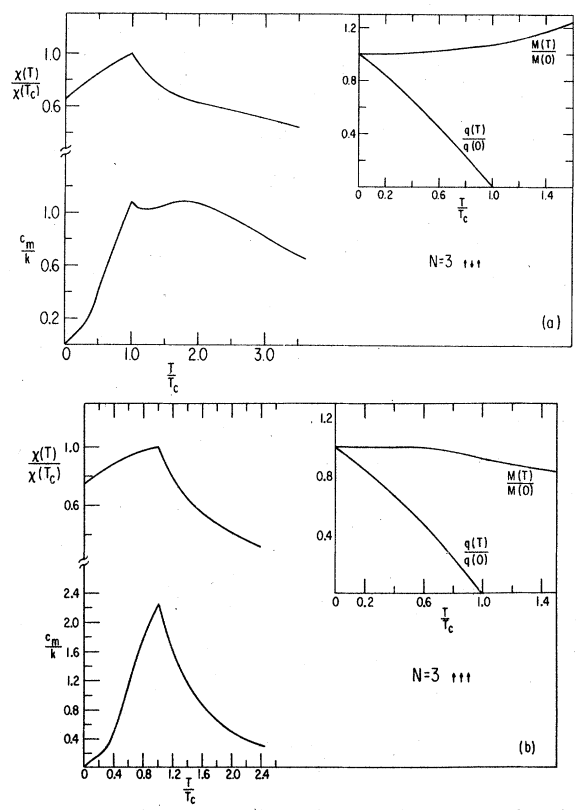

FIG. 1. Temperature dependences of the normalized static susceptibility χ and specific heat per cluster C_m for $N=3$ spins in a cluster with (a) antiferromagnetic and (b) ferromagnetic coupling. The inset plots the temperature dependences of the cluster moment M and order parameter q divided by their $T=0$ values. Other parameters are given in the text.

less symmetric than in the previous figure; χ is flatter over the entire interval $T \leq T_c$. The temperature dependence of M in the antiferromagnetic case is reflected most strongly in χ for $T > T_c$. Since M increases with increasing temperature, the susceptibility will decrease less rapidly above T_c than in the EA mean-field model. These effects are made less apparent by the presence of the third term in Eq. (2.10) which (because of its temperature dependence) serves to reduce the T variation of M below T_c in the ferromagnetic and above T_c in the antiferromagnetic case.

Figures 3 show the behavior of χ, C_m, q, and M as a function of temperature for $N=6$ and $R=0.32$. The spins were arranged so that they formed a right equilateral triangle (with three spins on the base and side). In this case, the ground-state moment $M(T=0)=2$. The behavior of all properties for the antiferromagnetic (a) and ferromagnetic cases (b) is similar to that of their counterparts for $N=3$ and 5.

Because of cost limitations we did not consider larger values of N. The behavior of χ and C_m for $N \leq 6$ suggests that for small N the antiferromagnetic case is in better accord with experiment than is the ferromagnetic one. However, as the value of N increases, C_m^{intra} grows so that for ferromagnetic clusters by $N=12$ we have observed a rounded maximum in C_m at a temperature $T_0 > T_c$ with a cusp in χ at T_c. These calculations, which were based on an Ising-model interaction (to reduce computational costs) will be published in a future paper.

Because our numerical results were qualitatively insensitive to the cluster size N, for reasonable values of this parameter, it is not possible to determine the mean cluster size from measurements of static properties, such as χ or C_m. To get more detailed information about N and the strengths and signs of intracluster interactions it is necessary

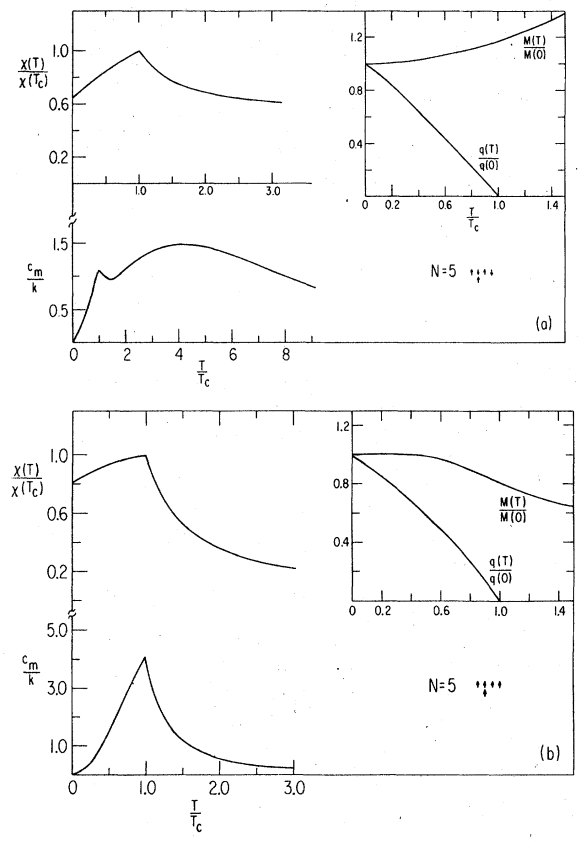

FIG. 2. Temperature dependences of the static susceptibility and specific heat per cluster for $N=5$ spins in an (a) antiferromagnetically and (b) ferromagnetically coupled cluster. The inset plots the temperature dependences of the normalized cluster moment and order parameter. The ground-state spin configuration is also shown.

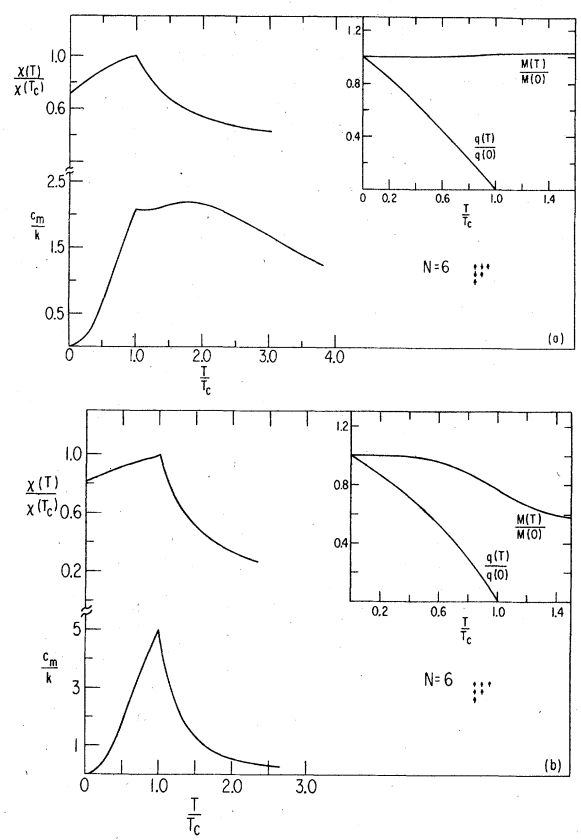

FIG. 3. Temperature dependences of the static susceptibility and specific heat per cluster for $N=6$ spins in an (a) antiferromagnetically and (b) ferromagnetically coupled cluster. The inset plots the temperature dependences of the normalized cluster moment and order parameter. The ground-state spin configuration is shown.

to look also at transport properties, such as the resistivity and neutron scattering cross section. The present cluster-mean-field theoretic approach can be readily extended to compute these dynamical properties. It should be noted that both transport measurements[9,29] suggest that clustering effects play an important role. In particular, that the maximum in the resistivity occurs at a temperature $T^{max} > T_c$ is evidence that there are "residual" internal magnetic fields above T_c (deriving, perhaps, from intracluster interactions); that the elastic neutron cross section $d\sigma/d\Omega_q$ peaks at a q dependent temperature has also been interpreted[29] as evidence for the importance of clusters in the spin glasses. Application of the CMFT to these transport properties will be discussed in a future publication.

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